

# Controlled Flow of Spin-Entangled Electrons via Adiabatic Quantum Pumping

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We propose a method to dynamically generate and control the flow of spin-entangled electrons, each belonging to a spin-singlet, by means of adiabatic quantum pumping. The pumping cycle functions by periodic time variation of localized two-body interactions. We develop a generalized approach to adiabatic quantum pumping as traditional methods based on scattering matrix in one dimension cannot be applied here. We specifically compute the flow of spin-entangled electrons within a Hubbard-like model of quantum dots, and discuss possible implementations and identify parameters that can be used to control the singlet flow.

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Entanglement is one of the most intriguing features that distinguish the quantum world from the classical. In recent years [1] applications potential in quantum information and computation has injected renewed vigor into the study of entangled states. Since Bohm's [2] reformulation of the Einstein-Podolsky-Rosen (EPR) paradox [3] in terms of pairing of spins, spin singlets have become the canonical example of an elementary entangled state. From an experimental standpoint, manipulating singlet pairs of electron spins, in particular, is promising because of the vast expertise already available in solid-state electronics [4].

Several proposals have emerged [5, 6, 7, 8] that aim to generate a controlled flow of electron spin singlets using Coulomb blockade and tunnelling at pinched quantum dots that have weak coupling to leads. In these proposals, the natural tunnelling of unwanted single electrons can be difficult to suppress and/or singlet delivery time is constrained by tunnelling rates that are slow due to the necessity for pinched dots. Since manipulations in a quantum information device need to occur before decoherence sets in, fast delivery is essential, and any background of unpaired electrons reduces the purity of entanglement. It is therefore highly desirable to have a fast dynamic mechanism to generate a selective flow only of electrons belonging to singlets. In this paper we show that this can be accomplished through a generalized adiabatic quantum pumping that is induced by localized interactions and we discuss possible ways of experimental realization. Pinched dot tunnelling times and singlet formation times do not constrain delivery rates in our proposal. In addition, the absence of bias and the adiabatic nature of the pumping can reduce the production of heat in the apparatus.

The notion of quantum pumping has its roots in a speculative paper by Thouless [9] in 1983, but advances in nanoscale transport have led to a renewed and growing interest in the phenomenon in recent years both theoretically and experimentally [10, 11, 12, 13, 14, 15, 16]. Quantum pumping is a coherent process that creates a direct current in the *absence* of any bias through a

nanoscale device, by changing its scattering properties periodically through independent adiabatic variation of two or more physical parameters. Adiabatic quantum pumping of charge [10], spin [13, 14] and thermal [15] currents have been considered. However previous studies have generally relied on a theoretical description based on transmission and reflection coefficients which cannot be used to describe the pumping of singlets due to localized interactions; therefore in this paper we also formulate a more generalized approach to the theory of quantum pumping.

*Singlet Current:* We consider a single available channel in a quasi one dimensional mesoscopic conductor connected to macroscopic contacts (Fig. 1(a)). In the presence of two body interactions the charge current can be defined in terms of the two particle reduced density matrix,  $\rho_2$ . If the interaction does *not* affect spins, the reduced density matrix separates into four independent spin subspaces, one singlet with a symmetric spatial part and three triplets with antisymmetric spatial parts; the singlet current is therefore equivalent to the charge current associated with the symmetric spatial part ( $\rho_2^S$ ) of

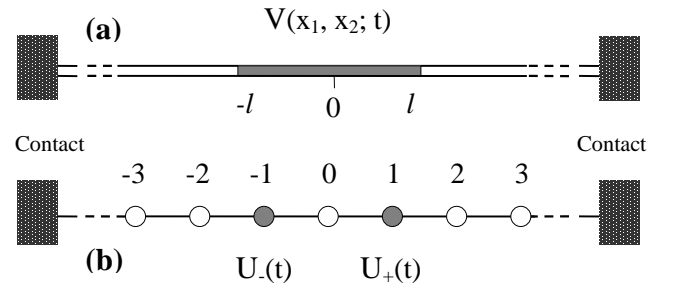


FIG. 1: (a) Schematic figure showing the *two-body* interaction  $V(x_1, x_2; t)$  acting in a finite interval  $(-l, l)$  in a one dimensional system, (b) Implementation in a tight-binding model where the two-body interaction is present only at lattice sites  $m = \pm 1$ ; the interaction strengths  $U_{\pm}(t)$  at the two sites are the time-dependent pumping parameters.

the two particle reduced density matrix

$$J_S(x_1, t) = \frac{e\hbar}{m} \int dx_2 \Im \{ \partial_{x_1} \rho_2^S(x_1, x_2; x'_1, x_2; t) \}_{x_1=x'_1}. \quad (1)$$

We consider a situation in which two-body interactions are localized, meaning that they are non-vanishing only

when the particles are in a certain finite interval, as shown in Fig. 1(a). Such interactions lend themselves to a scattering description and the current can be approximately evaluated by expanding the density matrix in terms of two particle scattering states

$$J_S(x_1, t) \simeq \frac{e\hbar}{2m} \int dE F(E) \int \frac{dk_1}{2\pi} \int \frac{dk_2}{2\pi} \delta\left(\frac{\hbar^2 k_1^2}{2m} + \frac{\hbar^2 k_2^2}{2m} - E\right) \int dx_2 \Im \{ \partial_{x_1} \Psi_{k_1, k_2}(x_1, x_2, t) \Psi_{k_1, k_2}^*(x'_1, x_2, t) \}_{x_1=x'_1}. \quad (2)$$

Here  $E$  denotes the energy required to remove a pair of particles from the many-body ground state, and  $F(E)$  is the distribution of this pair energy. The effect of interaction on the current is determined completely by the two-particle *singlet* scattering states,  $\Psi_{k_1, k_2}$  arising from *free* singlet states  $\Phi_{k_1, k_2}(x_1, x_2) = \frac{1}{\sqrt{2}}[\phi_{k_1}(x_1)\phi_{k_2}(x_2) + \phi_{k_1}(x_2)\phi_{k_2}(x_1)]$  where  $\phi_k(x)$  denotes a single particle plane wave state with momentum  $\hbar k$ .

*Pumped Current through Adiabatic Perturbation* : The two particle scattering states, and therefore the current, are determined by the interaction  $V(\bar{x}, t)$  between a pair of particles; we take it to be time-dependent and to occur only in a *finite* region  $|x_i| < l$ . Most importantly the interaction  $V(\bar{x}, t)$  is chosen to be localized so that it *only affects singlets thereby naturally eliminating the flow of triplets* in the absence of a bias. When the characteristic period  $\omega$  of the time variation of the potential is slow compared to the time  $\delta t$  the particles dwell in the scattering region [17],  $\omega \times \delta t \ll 1$ , we can apply adiabatic perturbation theory to express the scattering states of the time-dependent Hamiltonian in terms of the instantaneous states up to linear order

$$\Psi_{\bar{k}}(\bar{x}, t) \simeq \Psi_{\bar{k}}^t(\bar{x}) - i\hbar \int d\bar{x}' G^t(\bar{x}, \bar{x}'; E) \frac{\partial}{\partial t} \Psi_{\bar{k}}^t(\bar{x}'). \quad (3)$$

We use the notation  $\bar{x} \equiv \{x_1, x_2\}$  and  $\bar{k} \equiv \{k_1, k_2\}$ , so that  $G^t(\bar{x}, \bar{x}'; E)$  is the *two-particle instantaneous* retarded Green's function for the *full* Hamiltonian. The instantaneous state  $\Psi_{\bar{k}}^t(\bar{x})$  is a solution of the *time-independent* Lippmann-Schwinger equation for the potential  $V(\bar{x}', t)$  at the specific time  $t$ ,

$$\Psi_{\bar{k}}^t(\bar{x}) = \Phi_{\bar{k}}(\bar{x}) + \int d\bar{x}' G_0(\bar{x}, \bar{x}'; E) V(\bar{x}', t) \Psi_{\bar{k}}^t(\bar{x}') \quad (4)$$

where  $G_0$  is the *free two-particle retarded* Green's func-

tion. Taking the time derivatives of the defining equations for  $\Psi^t(\bar{x}')$  and  $G^t(\bar{x}, \bar{x}'; E)$  enables us to express the second term in Eq.(3), that is linear in  $\partial_t$ , as

$$\Delta \Psi_{\bar{k}}(\bar{x}, t) = -i\hbar \int d\bar{x}' \int d\bar{x}'' G^t(\bar{x}, \bar{x}'; E) \times G^t(\bar{x}', \bar{x}''; E) \dot{V}(\bar{x}'', t) \Psi_{\bar{k}}^t(\bar{x}''). \quad (5)$$

If there is no bias or time-dependence, the laws of thermodynamics demand that there should be no current; we explicitly confirm that our expression for the current satisfies this essential physical requirement. The net current in the absence of time-dependence is evaluated by using the zeroeth order term from Eq. (3) for the scattering state in Eq. (2):  $\Psi_{k_1, k_2}(x_1, x_2, t) \equiv \Psi_{k_1, k_2}^t(x_1, x_2)$ . The resulting expression can be simplified by relating the imaginary part of the retarded Green's function to the free singlet states:  $\Im\{G_0(\bar{x}, \bar{x}'; E)\} = -\pi \int dk_1 \int dk_2 \delta\left(\frac{\hbar^2 k_1^2}{2m} + \frac{\hbar^2 k_2^2}{2m} - E\right) \Phi_{\bar{k}}(\bar{x}) \Phi_{\bar{k}}^*(\bar{x}')$ . Then repeated use of the Lippmann-Schwinger equation and properties of the Green's functions shows that the net current corresponding to the zeroeth order  $\Psi^t(\bar{x})$  vanishes.

After confirming that our expression cannot produce spontaneous current, we evaluate the singlet current induced by the adiabatic time evolution. To linear order in the time dependence this involves the evaluation of

$$\int dx_2 \Im \{ \partial_{x_1} \Psi_{\bar{k}}^t(x_1, x_2) \Delta \Psi_{\bar{k}}^*(x'_1, x_2, t) \}_{x_1=x'_1} \quad (6)$$

within the expression for the current in Eq. (2). A calculation employing standard Green's function identities, similar to that for the zeroeth order, leads to an expression for the net amount of singlet entangled electron pairs pumped in a complete cycle of period  $\tau$ , yield the main result of this paper:

$$Q_S(\tau) = \frac{e\hbar^2}{2\pi m} \int_0^\tau dt \int dE F(E) \frac{\partial}{\partial E} \left[ \int d\bar{x}' \dot{V}(\bar{x}', t) \int dx_2 \Im \{ G^{t*}(\bar{x}, \bar{x}'; E) \partial_{x_1} G^t(\bar{x}, \bar{x}'; E) \} \right]. \quad (7)$$

*Singlet Pumping in a Turnstile Model:* We illustrate our results with a tight-binding model, (Fig. 1(b)) with two Hubbard impurities located at sites  $-m, m$

$$V(\bar{n}, t) = U_-(t)\delta_{n_1, -m}\delta_{n_2, -m} + U_+(t)\delta_{n_1, m}\delta_{n_2, m}. \quad (8)$$

An electron can interact with another only at those two sites therefore, due to the Pauli principle, only singlets are affected. The strength of the interactions,  $U_{\pm}(t)$  are the two time-dependent pumping parameters. This concept is similar to a ‘turnstile model’ [11] but differs sig-

nificantly in that, instead of time-varying external potentials, the two-body interaction among electrons is varied in time. We separately derived a discrete version of Eq. (7); the end result amounts to replacing the coordinate arguments with site indices  $x \rightarrow n$ , integrals by sums and derivatives with a finite difference form. A lengthy calculation leads to an expression for the singlets pumped in a complete cycle in terms of the *free two-particle lattice* Green’s function, specifically two of its matrix elements  $G_0(0) = G_0(\bar{m}, \bar{m}; E)$  and  $G_0(2\bar{m}) = G_0(\bar{m}, -\bar{m}; E)$

$$Q_S(\tau) = \frac{-e}{2\pi} \int_0^\tau dt \int dE F(E) \frac{\partial}{\partial E} \sum_{\pm} \dot{U}_{\pm}(t) \frac{|T_{\pm}(t)|^2 [\Im\{G_0(0)\}(1 + |T_{\mp}(t)G_0(2\bar{m})|^2) \pm 2\Im\{T_{\mp}(t)G_0(2\bar{m})G_0^{\pm}(2\bar{m})\}]}{U_{\pm}(t)^2 |1 - T_{\mp}(t)T_{\pm}(t)G_0(2\bar{m})G_0(2\bar{m})|^2}. \quad (9)$$

Here  $\bar{m} \equiv \{m, m\}$ , and  $T_{\pm}(t) = 1/(U_{\pm}^{-1}(t) + G_0(0))$  is the T-matrix for a single Hubbard impurity, and  $G_0^{\pm} \equiv G_0(2\bar{m})$  for  $\lambda = \pm i|\lambda|$ , when expressed in the form  $G_0(2\bar{m}) = \int_0^\pi \frac{dk}{2\pi} \frac{e^{2m(ik-\lambda)}}{\sinh(\lambda)}$  with  $\cosh(\lambda) = E/2 - \cos(k)$ .

Exact analytical forms exist for the lattice Green’s functions,  $G_0$ , in terms of elliptic integrals [18]. For the purpose of numerical estimates, we assume a square-profile time dependence [12] in the plane of the parameters  $U_{\pm}(t)$ , shown in Fig. 2(a), where the two parameters change alternately between minimum value  $U_{min}$  and a maximum value  $U_{max}$ . The pair distribution function is taken to be a Fermi function,  $F(E) \simeq 1/[e^{\beta(E-\mathcal{E})} + 1]$ . At low temperatures, an integration by parts with respect to energy yields an expression for the pumped singlets in terms of the maximum energy,  $\mathcal{E}$  available for a pair.

The onsite energy of each tight-binding site is taken to be zero and all energies are expressed in units of the

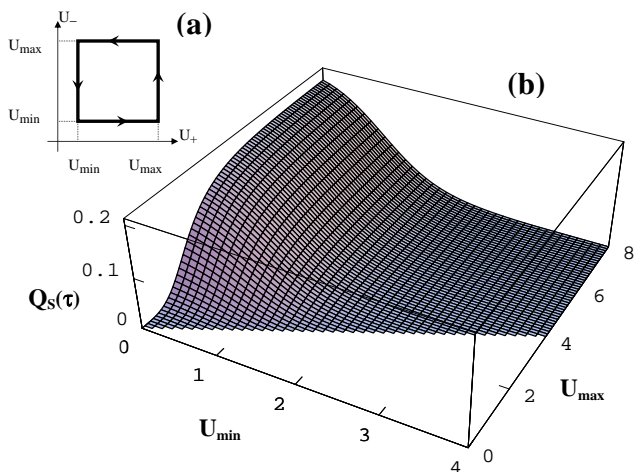


FIG. 2: (a) Pumping cycle in the space of parameters  $U_{\pm}$ . (b) Net singlets pumped in a single cycle as a function of the location and size of the square pumping cycle as  $U_{min}$  and  $U_{max}$  are varied.

nearest neighbor coupling strength. In Fig. 2(b) we plot the net singlets pumped in a single cycle as a function of the size and location of the square footprint of the time-cycle in the space of the parameters  $U_{\pm}$ ; the flow depends on the enclosed region. Figure 3 shows the dependence of the singlet current on the parameter  $\mathcal{E}$  that measures the available energy for pairs of electrons determined by the chemical potential in the contacts. The two curves in the figure correspond to different locations of the Hubbard impurities, at lattice sites  $m = \pm 1$  and  $m = \pm 2$ , illustrating the significant effect the spatial separation of two impurities have on the pumping rate. The direction of flow can also reverse for certain values of the various parameters; reversing the time-cycle is not the only way to reverse the direction of the current [10]. Quantum pumping has the intrinsic property that the magnitude of the pumped quantity, in this case singlets, is continuous in nature so that the delivery rate per cycle can be continuously adjusted. Thus there are several ways to precisely control the magnitude and direction of the flow of singlets dynamically.

*Discussion and Outlook:* The turnstile model could be implemented by taking the interaction sites to be quantum dots coupled to leads and varying the electron-electron interaction by decreasing the size of the dot periodically. Concurrent variation in the dot-lead coupling could in principle be compensated by counter-varying gate voltages. Another way to vary the interaction strength is to change the dielectric constant locally. This could be accomplished by a local shift of the electron wavefunction among layers of different materials in a heterostructure; this technique was recently used to control the electron  $g$  factor [19]. A shift between silicon and germanium layers or gallium arsenide (GaAs) and aluminium arsenide (AlAs) layers can in principle produce up to 25% change in the dielectric constant, sufficient to

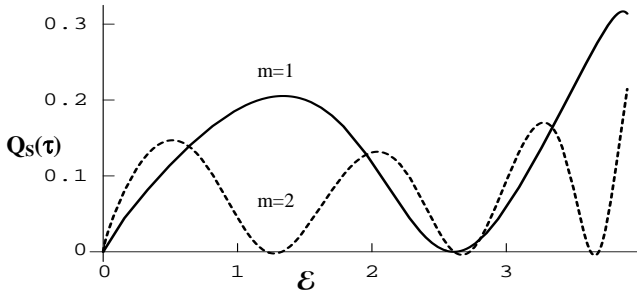


FIG. 3: Net singlets pumped per cycle as a function of the maximum available energy  $\mathcal{E}$  for pairs. The two curves correspond to different separations of the Hubbard impurities, solid line  $\rightarrow \pm m = \pm 1$  and dashed line  $\rightarrow \pm m = \pm 2$ .

see the effects described here.

The adiabatic condition requires the period,  $\tau \gg \delta t$ , the dwell time given by  $\delta t = d/v$ , with  $d$  being the size of the scattering region and  $v$  the carrier velocity [17]. Since our singlet pump requires no potential barriers that reduce kinetic energy,  $v$  may be taken to be the Fermi velocity typically  $> 10^5$  m/s, so for a scattering region  $d \sim 10$  nanometers the adiabatic condition would allow thousands of cycles per nanosecond. Our numerical estimates then give a pumping rate at the order of a thousand singlets per nanosecond. In typical Coulomb-blockade based schemes the most optimistic estimates yield a delivery rate at the order of one nanosecond per singlet [6, 7]. Thus our approach has the potential to be much faster.

Our result, Eq. (7), has the merit that it can also be applied to quantum pumping in systems that allow an independent particle description, as considered in previous studies. All the elements in Eq. (7) then reduce to single particle functions:  $F(E) \rightarrow f(E)$  is the Fermi distribution function, the Green's function is a *single* particle one with an asymptotic form  $\rightarrow -i[m/(\hbar^2 k)]e^{ikx}\psi_k^{t*}(x')$  in terms of the scattering state  $\psi_k^t(x')$  and wavevector  $k = [2mE/\hbar^2]^{-1/2}$ . This yields the pumped *charge*

$$Q(\tau) = \frac{em}{2\pi\hbar^2} \int_0^\tau dt \int dE f(E) \frac{\partial}{\partial E} \left\{ \frac{1}{k} \langle \psi^t | \dot{V} | \psi^t \rangle \right\}, \quad (10)$$

which agrees with expressions derived in earlier works [10, 12]. But unlike all previous treatments, we never use 1D scattering matrix elements as they do not have useful generalizations when particles interact with each other in a region rather than scatter off an external potential.

To summarize, we have proposed a method based on two-body adiabatic quantum pumping for generating a dynamically controlled flow of spin-entangled electrons. The process is inherently coherent, potentially much faster than most current proposals, has reduced noise because of the lack of bias and the natural elimination of single electron flow, and allows for continuous adjustment of the flow through numerous physical parameters. All of these features can be developed and incorporated into

a comprehensive scheme to generate a controlled flow of entangled electrons. Our goal here has been to present the basic idea, develop a theoretical framework for its description and discuss a possible physical model for implementation. We have in the process generalized the treatment of quantum pumping to incorporate interactions that cannot be treated in a scattering matrix approach. We anticipate that future studies can build upon the considerations in this work.

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*Note added:* When we were finishing this paper we become aware of an excellent recent proposal by Beenakker *et al.* [20] to excite entangled electron-hole pairs using one-body potential in a scattering matrix approach. Our proposal is more difficult to implement, but it could be naturally incorporated into an electron-only spintronic device, it produces a current of singlets unadulterated by single electrons, and it will enjoy a longer decoherence time since hole decoherence times tend to be short.

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- [1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, (Cambridge, New York, 2000).
  - [2] D. Bohm, *Quantum Theory*, (Prentice-Hall, New Jersey, 1951).
  - [3] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
  - [4] *Semiconductor Spintronics and Quantum Computation*, edited by D.D. Awschalom, D. Loss, and N. Samarth (Springer, 2002).
  - [5] W. D. Oliver, F. Yamaguchi, and Y. Yamamoto, Phys. Rev. Lett. **88**, 037901 (2002).
  - [6] D. S. Saraga and D. Loss, Phys. Rev. Lett. **90**, 166803 (2003).
  - [7] X. Hu and S. Das Sarma, Phys. Rev. B **69**, 115312 (2004).
  - [8] P. Recher, E. V. Sukhorukov, D. Loss Phys. Rev. B **63**, 165314 (2001).
  - [9] D. J. Thouless, Phys. Rev. B **27**, 6083 (1983).
  - [10] P. W. Brouwer, Phys. Rev. B **58**, R10135 (1998).
  - [11] Y. Levinson, O. Entin-Wohlman, and P. Wölfle, Physica A **302**, 335 (2001).
  - [12] O. Entin-Wohlman, A. Aharony, and Y. Levinson, Phys. Rev. B **65**, 195411 (2002).
  - [13] P. Sharma and C. Chamon, Phys. Rev. Lett. **87**, 096401 (2001).
  - [14] S. K. Watson, R. M. Potok, C. M. Marcus, and V. Umansky, Phys. Rev. Lett. **91**, 258301 (2003).
  - [15] M. Moskalets and M. Büttiker, Phys. Rev. B **66**, 205320 (2002).
  - [16] S. W. Kim, Phys. Rev. B **66**, 235304 (2002).
  - [17] M. Büttiker and R. Landauer, Phys. Rev. Lett. **49**, 1739 (1982).
  - [18] E. N. Economou, *Green's Functions in Quantum Physics*, (Springer-Verlag, Berlin 1979).
  - [19] H. W. Jiang and E. Yablonovitch, Phys. Rev. B **64**, 041307 (2001).
  - [20] C. W. J. Beenakker, M. Titov, and B. Trauzettel, Phys.

Rev. Lett. **94**, 186804 (2005).